A Lightweight Implementation of a Shuffle Proof for Electronic Voting Systems

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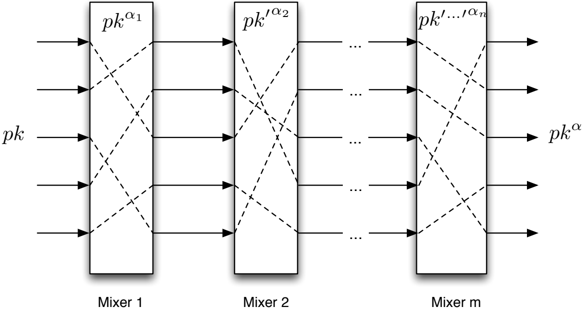
UniVote: Verifiable Electronic Voting over the Internet

- Internet voting system for student board elections at Swiss universities
- Project started in 2012
- First elections in spring 2013
- 6 elections were held successfully
- https://www.univote.ch

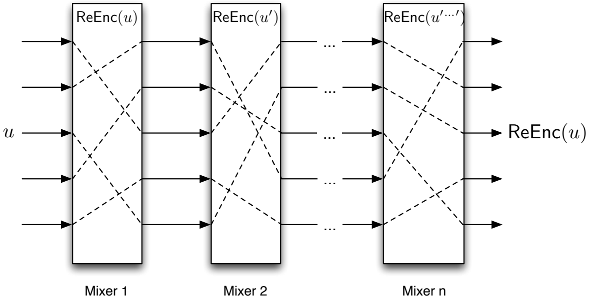
UniVote is (not yet) end-to-end verifiable and offers anonymized vote casting [Neff01, HS11].

Mixing of public keys:



Before the final decryption and tally phase, the ballots are mixed.

- Late registration (students cannot be forced to register before voting phase)
- Anonymous channel cannot always be expected
- No performance issue (only a few thousand ballots)



UniCrypt is a cryptographic Java library:

- Simplifies the implementation of cryptographic voting protocols
- Split into two layers: mathematical fundament and cryptographic primitives
- Type safety on a mathematical level
- https://github.com/bfh-evg/unicrypt

Mix-Net Implementations

- Verificatum: An implementation of a full-featured mix-net by Wikström
- A number of prototype implementations of shuffle proofs

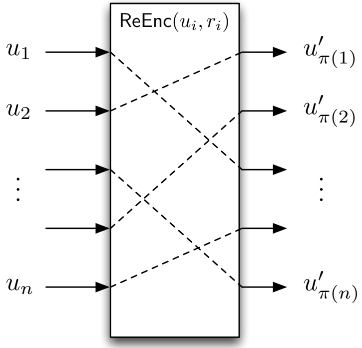
Contribution

A new implementation of a shuffle proof

- Based on the findings of Wikström and Terelius [Wik09, TW10]
- Embedded in a cryptographic library with a clean and intuitive application programming interface
- Full flexibility with respect to the encryption system and the algebraic groups
- Support for different types of mix-nets
- Portable to any device running a Java Virtual Machine

Proof of a (Re-Encryption) Shuffle

Proof that each ciphertext of a list of ciphertexts has been re-encrypted and permuted



Wikström/Terelius's Shuffle Proof

[**Wik09**] A Commitment-Consistent Proof of a Shuffle

- Offline part: Commit to a permutation matrix and proof that it is indeed a permutation matrix.
- Online part: Shuffle the input batch and give a commitment-consistent proof of a shuffle.

[**TW10**] Proofs of Restricted Shuffles

- Restricting the set of permutations.
- A new proof of a shuffle based on a permutation matrix.

Implementation

Shuffling and the shuffle proof are implemented in UniCrypt inside the following cryptographic components:

Mixer Covers the shuffle functionality without proving its correctness. Two implementations: *re-encryption mixer* and *identity mixer*

Proof System Holds all types of zero-knowledge proofs, including the proof of a shuffle

Challenge Generator Creates challenges in an interactive or non-interactive manner

Example of Usage

```
// Select cyclic group for safe prime p=2q+1 (1024 bit)
CyclicGroup group = GStarModSafePrime.getRandomInstance(1024);

// Create ElGamal encryption scheme and key pair
ElGamalEncryptionScheme elGamal =
    ElGamalEncryptionScheme.getInstance(group);

Pair keys = elGamal.getKeyPairGenerator().generateKeyPair();
Element pk = keys.getSecond();

// Set shuffle size and create random ElGamal ciphertexts
int n = 100;
Tuple ciphertexts = Tuple.getInstance();
for (int i = 0; i < n; i++) {
    Element m = group.getRandomElement();
    Pair c = elGamal.encrypt(pk, m);
    ciphertexts = ciphertexts.add(c);
}
```

Listing 1: Setup

Example of Usage

```
// Create mixer, random permutation pi, and randomizations r
ReEncryptionMixer mixer =
    ReEncryptionMixer.getInstance(elGamal, pk, n);

PermutationElement pi =
    mixer.getPermutationGroup().getRandomElement();

Tuple r = mixer.generateRandomizations();

// Shuffle ciphertexts using pi and r
Tuple shuffledCiphertexts = mixer.shuffle(ciphertexts, pi, r);
```

Listing 2: Shuffle

Example of Usage

```
// Create permutation commitment c_pi based on pi
// and randomizations s
PermutationCommitmentScheme pcs =
    PermutationCommitmentScheme.getInstance(group, n);
Tuple s = pcs.getRandomizationSpace().getRandomElement();
Tuple c_pi = pcs.commit(pi, s);

// Create permutation commitment proof system
PermutationCommitmentProofSystem pcps =
    PermutationCommitmentProofSystem.getInstance(group, n);

// Define private and public input
Pair offlinePrivateInput = Pair.getInstance(pi, s);
Element offlinePublicInput = c_pi;

// Generate permutation commitment proof
Pair offlineProof =
    pcps.generate(offlinePrivateInput, offlinePublicInput);
```

Listing 3: Online Phase (Proof of Knowledge of Permutation Matrix)

Example of Usage

```
// Create shuffle proof system
ReEncryptionShuffleProofSystem rsps =
    ReEncryptionShuffleProofSystem
        .getInstance(group, n, elGamal, pk);

// Define private and public input
Triple onlinePrivateInput = Triple.getInstance(pi, s, r);
Triple onlinePublicInput =
    Triple.getInstance(c_pi, ciphertexts, shuffledCiphertexts);

// Generate shuffle proof
Triple onlineProof =
    rsps.generate(onlinePrivateInput, onlinePublicInput);
```

Listing 4: Online Phase (Commitment Consistent Proof of a Shuffle)

Example of Usage

```
// Verify permutation commitment proof
boolean v1 = pcps.verify(offlineProof, offlinePublicInput);

// Verify shuffle proof
boolean v2 = rsps.verify(onlineProof, onlinePublicInput);

// Verify equality of permutation commitments
boolean v3 =
  offlinePublicInput.isEquivalent(onlinePublicInput.getFirst());

if (v1 && v2 && v3) success();
```

Listing 5: Proof Verification

Thank you!

Wikström/Terelius's Shuffle Proof

An $N \times N$ - matrix M is a permutation matrix if there is exactly one non-zero element in each row and column and if this non-zero element is equal to one.

Example:

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_3 \\ x_1 \\ x_2 \end{pmatrix}$$

If M_π is a permutation matrix for the permutation π then

$$M_\pi \cdot \bar{x} = \bar{x}' = (x_{\pi(1)}, \ldots, x_{\pi(N)})$$

Wikström/Terelius's Shuffle Proof

Theorem (Permutation Matrix) [TW10]
Let $M = (m_{i,j})$ be an $N \times N$ - matrix over $\mathbb{Z}_q$ and $\bar{x} = (x_1, \ldots, x_N)$ be a list of variables. Then M is a permutation matrix if and only if

$$\prod_{i=1}^{N} \langle \bar{m}_i, \bar{x} \rangle = \prod_{i=1}^{N} x_i \quad \text{and} \quad M\bar{1} = \bar{1}$$

m_i denotes the i-th row vector of M and $\langle \bar{m}_i, \bar{x} \rangle = \sum_{j=1}^{N} m_{i,j}\, x_j$

Wikström/Terelius's Shuffle Proof

A matrix commitment based on the generalized Pedersen commitment has the property:

$$\langle Com(M, \bar{s}), \bar{e} \rangle = Com(M\bar{e}, \langle \bar{s}, \bar{e} \rangle)$$

It follows that if M is a permutation matrix then $M\bar{e} = \bar{e}' = (e_{\pi(1)}, \ldots, e_{\pi(N)})$ and $\langle Com(M, \bar{s}), \bar{e} \rangle$ is a publicly computed commitment to the permuted $\bar{e}$ - vector based on the commitment to M.

Proof of Knowledge of Permutation Matrix (offline) 1/2

Common Input: Matrix commitment c_π
Private Input: Permutation matrix M_π and $\bar{s}$ such that $c_\pi = Com(M_\pi, \bar{s})$.

1. $\mathcal{V}$ chooses $\bar{e} \in \mathbb{Z}_q^N$ randomly and hands $\bar{e}$ to $\mathcal{P}$
2. $\mathcal{P}$ computes $v = \langle \bar{s}, \bar{1} \rangle$, $w = \langle \bar{s}, \bar{e} \rangle$ and $\bar{e}' = M_\pi \bar{e}$.
3. $\mathcal{V}$ outputs the result of

$$\Sigma\text{-proof}\left[\begin{matrix} v, w \in \mathbb{Z}_q \\ \bar{e}' \in \mathbb{Z}_q^N \end{matrix} \;\middle|\; Com(\bar{1}, v) = \langle c_\pi, \bar{1} \rangle \wedge Com(\bar{e}', w) = \langle c_\pi, \bar{e} \rangle \wedge \prod_{i=1}^{N} e'_i = \prod_{i=1}^{N} e_i \right]$$

Proof of Knowledge of Permutation Matrix (offline) 2/2

The Σ-proof of the proof of knowledge of permutation matrix can be transformed into a generic preimage proof by the homomorphic one-way function:

$$\phi_{offline}(v, w, \bar{t}, d, \bar{e}') = \Big(Com(\bar{1}, v), Com(\bar{e}', w), g^{t_1} c_0^{e'_1}, \ldots, g^{t_N} c_{N-1}^{e'_N}, Com(0, d)\Big)$$

With additional private input: Randomness $\bar{t} \in \mathbb{Z}_q^N$ and $d = d_N$ and $d_i = t_i + e'_i d_{i-1}$ for $i > 2, \ldots, N$ with $d_1 = t_1$. $c_i = g^{t_i} c_{i-1}^{e'_i}$ and $c_0 = h$.

Commitment-Consistent Proof of a Shuffle (online) 1/2

Common Input: Permutation matrix commitment c_π and ciphertexts (ElGamal) $u_1, \ldots, u_N, u'_1, \ldots, u'_N \in (G_q \times G_q)$.
Private Input: Permutation π and randomness $\bar{r} \in \mathbb{Z}_q^N$ such that $u'_i = ReEnc(u_{\pi(i)}, r_{\pi(i)})$.

1. $\mathcal{V}$ chooses $\bar{e} \in \mathbb{Z}_q^N$ randomly and hands $\bar{e}$ to $\mathcal{P}$
2. $\mathcal{P}$ computes $w = \langle \bar{s}, \bar{e} \rangle$, $r = \langle \bar{r}, \bar{e} \rangle$ and $\bar{e}' = M_\pi \bar{e}$.
3. $\mathcal{V}$ outputs the result of

$$\Sigma\text{-proof}\left[\begin{matrix} r, w \in \mathbb{Z}_q \\ \bar{e}' \in \mathbb{Z}_q^N \end{matrix} \;\middle|\; Com(\bar{e}', w) = \langle c_\pi, \bar{e} \rangle \wedge \prod_{i=1}^{N} (u'_i)^{e'_i} = ReEnc(\prod_{i=1}^{N} (u_i)^{e_i}, r)\right]$$

Commitment-Consistent Proof of a Shuffle (online) 2/2

The Σ-proof of the proof of knowledge of permutation matrix can be transformed into a generic preimage proof by the homomorphic one-way function:

$$\phi_{online}(r, w, \bar{e}') = \left(Com(\bar{e}', w), \prod_{i=1}^{N} (u_i')^{e_i'} Enc(1, -r) \right)$$