Batch Zero-Knowledge Proofs

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Outline

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Motivation

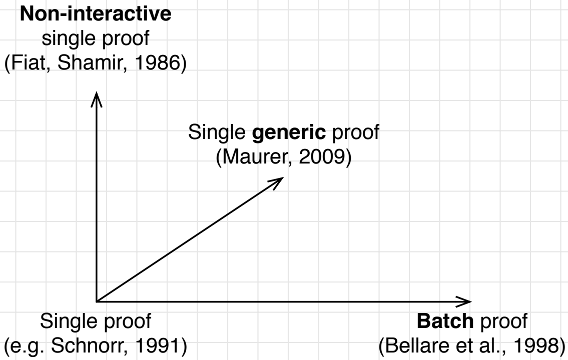
Proving multiple instances of the same type of zero-knowledge proof can be done in two ways

1. Standard AND-composition proof
   - → Linear proof size (commitment, response)
   - → Linear running time (generation, verification)
2. Batch Proof
   - → Constant proof size
   - → Improved running time

Applications

- Multi-decryption of ciphertexts $c_1, \ldots, c_n$
  - → $c_i = (a_i, b_i) =$ ElGamal ciphertext
  - → $m_i = b_i \cdot a_i^{-x} =$ decryption with private key x
- Multi-committment to messages $m_1, \ldots, m_n$ (e.g. commiting to a matrix)
  - → $m_i = i$-th column vector of M
  - → $c_i = C(m_i, s_i) =$ extended Pedersen commitment of m_i
- Multi-blinding values $x_1, \ldots, x_n$
  - → $z =$ common blinding value
  - → $x_i' = x_i^z =$ blinding of x_i
- Commitment multiplication proof of length > 2

Overview


Non-interactive
single proof
(Fiat, Shamir, 1986)
Single **generic** proof
(Maurer, 2009)
Single proof
(e.g. Schnorr, 1991)
Batch proof
(Bellare et al., 1998)

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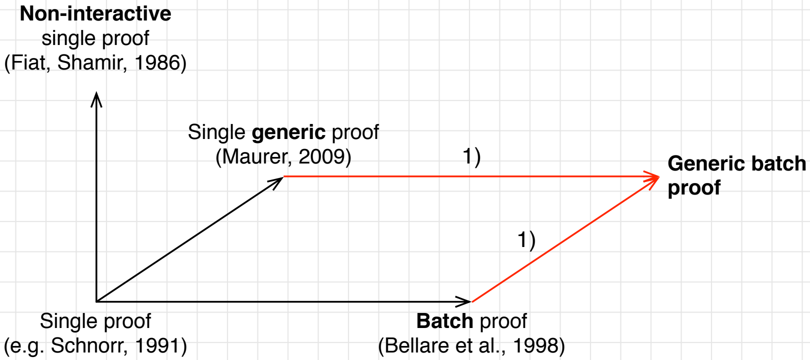
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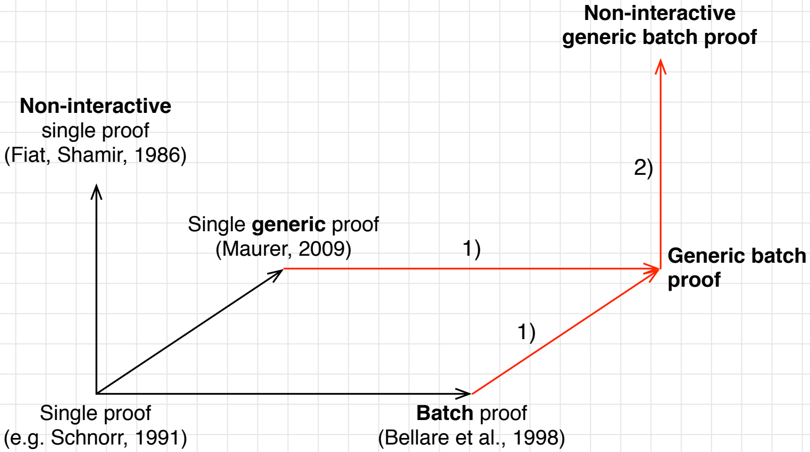
Overview



Non-interactive
generic batch proof
Non-interactive
single proof
(Fiat, Shamir, 1986)
2)
Single **generic** proof
(Maurer, 2009)
1)
Generic batch
proof
1)
Single proof
(e.g. Schnorr, 1991)
Batch proof
(Bellare et al., 1998)

References

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Outline

Non-Interactive Preimage Proof

- Let $(X, +, 0)$ and $(Y, \times, 1)$ be groups of finite order
- Consider a one-way group homomorphism $\phi : X \to Y$
- Let $b = \phi(a)$ be publicly known
- The prover P proves knowledge of a using the Σ-protocol:
  1. Choose $\omega \in_R X$ uniformly at random
  2. Compute $t = \phi(\omega)$
  3. Compute $c = H(b, t)$
  4. Compute $s = \omega + c \cdot a$
  5. Publish $\pi = (t, s)$
- To verify π, the verifier V computes $c = H(b, t)$ and checks $\phi(s) \stackrel{?}{=} t \cdot b^c$

Outline

Example 1: Discrete Logarithm (Schnorr)

- Let g be a generator of G_q
- Let $c = g^m$ be a publicly known commitment of $m \in \mathbb{Z}_q$
- P proves knowledge of m using the Σ-protocol for:

$$a = m,$$
$$b = c,$$
$$\phi(x) = g^x,$$

where $\phi : \underbrace{\mathbb{Z}_q}_{X} \to \underbrace{G_q}_{Y}$

Example 2: Pedersen Commitment

- Let g and h be generators of G_q
- Let $c = g^m h^s$ be a publicly known Pedersen commitment of $m \in \mathbb{Z}_q$ with randomization $s \in \mathbb{Z}_q$
- P proves knowledge of m and s using the Σ-protocol for:

$$a = (m, s),$$
$$b = c,$$
$$\phi(x_1, x_2) = g^{x_1} h^{x_2},$$

where $\phi : \underbrace{\mathbb{Z}_q \times \mathbb{Z}_q}_{X} \to \underbrace{G_q}_{Y}$

- Note that $\omega = (\omega_1, \omega_2)$ and $s = (s_1, s_2)$, but t is a single value

Example 3: Equality of Discrete Logarithms

- Let g_1 and g_2 be generators of G_q
- Let $c_1 = g_1^m$ and $c_2 = g_2^m$ be public commitments of $m \in \mathbb{Z}_q$
- P proves knowledge of m using the Σ-protocol for:

$$a = m,$$
$$b = (c_1, c_2),$$
$$\phi(x) = (g_1^x, g_2^x),$$

where $\phi : \underbrace{\mathbb{Z}_q}_{X} \to \underbrace{G_q \times G_q}_{Y}$

- Note that $t = (t_1, t_2)$, but ω and s are single values

Outline

Composed Proofs

- Consider n one-way group homomorphisms $\phi_i : X_i \to Y_i$
- Let $b_1, \ldots, b_n$ be publicly known, where $b_i = \phi_i(a_i)$
- P proves knowledge of $a_1, \ldots, a_n$ using the Σ-protocol for:

$$
\begin{aligned}
a &= (a_1, \ldots, a_n), \\
b &= (b_1, \ldots, b_n), \\
\phi(x_1, \ldots, x_n) &= (\phi_1(x_1), \ldots, \phi_n(x_n)),
\end{aligned}
$$

where $\phi : \underbrace{X_1 \times \cdots \times X_n}_{X} \to \underbrace{Y_1 \times \cdots \times Y_n}_{Y}$

- Note that $\omega = (\omega_1, \ldots, \omega_n)$, $t = (t_1, \ldots, t_n)$, $s = (s_1, \ldots, s_n)$

Composed Proofs: Performance

- Proof $\pi = (t, s)$ has size $O(n)$
- Generation: $O(n)$
  - → Let $\bar{r}$ be the average number of modExps in ϕ_i
  - → Computing $\phi(\omega)$ requires $n \cdot \bar{r}$ modExps
  - → $modExps(n) = n \cdot \bar{r}$
- Verification: $O(n)$
  - → Computing $\phi(s)$ requires $n \cdot \bar{r}$ modExps
  - → Computing b^c requires n modExps
  - → $modExps(n) = n \cdot (\bar{r} + 1)$
- Remark: c is usually small (e.g., 160 bits for SHA-1)

Special Case 1: Common Function

- Consider a single one-way group homomorphism $\phi : X \to Y$
- Let $b_1, \ldots, b_n$ be publicly known, where $b_i = \phi(a_i)$
- P proves knowledge of $a_1, \ldots, a_n$ using the Σ-protocol for:

$$a = (a_1, \ldots, a_n),$$
$$b = (b_1, \ldots, b_n),$$
$$\phi(x_1, \ldots, x_n) = (\phi(x_1), \ldots, \phi(x_n)),$$

where $\phi : \underbrace{X \times \cdots \times X}_{X} \to \underbrace{Y \times \cdots \times Y}_{Y}$

- Note that $\omega = (\omega_1, \ldots, \omega_n)$, $t = (t_1, \ldots, t_n)$, $s = (s_1, \ldots, s_n)$
- Proof size and performance remain unchanged

Special Case 2: Common Preimage

- Consider n one-way group homomorphisms $\phi_i : X \to Y$
- Let $b_1, \ldots, b_n$ be publicly known, where $b_i = \phi_i(a)$
- P proves knowledge of a using the Σ-protocol for:

$$\begin{aligned} & a \\ & b = (b_1, \ldots, b_n), \\ & \phi(x) = (\phi_1(x), \ldots, \phi_n(x)), \end{aligned}$$

where $\phi : X \to \underbrace{Y \times \cdots \times Y}_{Y}$

- Note that $t = (t_1, \ldots, t_n)$, but ω and s are single values
- Improved (linear) proof size, performance remains unchanged

Outline

Outline

Type-1 Batch Proof

- Consider a single one-way group homomorphism $\phi : X \to Y$
- Let $b_1, \ldots, b_n$ be publicly known, where $b_i = \phi(a_i)$
- P proves knowledge of $a_1, \ldots, a_n$ as follows:
  - → V chooses $e_1, \ldots, e_n$ uniformly at random

$$b = \prod_i b_i^{e_i} = \prod_i \phi(a_i)^{e_i} = \prod_i \phi(e_i a_i) = \phi(\sum_i e_i a_i)$$

  - → P generates a preimage proof $\pi = (t, s)$ for $a = \sum_i e_i a_i$ and $b = \phi(a)$
- V computes $b = \prod_i b_i^{e_i}$ and verifies π
- Note that P does not need to compute $b = \prod_i b_i^{e_i}$

Type-1 Batch Proof: Non-Interactive Version

- Consider a single one-way group homomorphism $\phi : X \to Y$
- Let $b_1, \ldots, b_n$ be publicly known, where $b_i = \phi(a_i)$
- P proves knowledge of $a_1, \ldots, a_n$ as follows:
  1. Choose $\omega \in_R X$ uniformly at random
  2. Compute $t = \phi(\omega)$
  3. Compute $e_i = H(b_i, t)$ for $i = 1, \ldots, n$
  4. Compute $a = \sum_i e_i a_i$
  5. Compute $c = H(b, t)$
  6. Compute $s = \omega + c \cdot a$
  7. Publish $\pi = (t, s)$
- V computes $e_i = H(b_i, t)$, $c = H(b_1, \ldots, b_n, t)$, $b = \prod_i b_i^{e_i}$, and checks $\phi(s) \stackrel{?}{=} t \cdot b^c$

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- Let $b_1, \ldots, b_n$ be publicly known, where $b_i = \phi(a_i)$
- P proves knowledge of $a_1, \ldots, a_n$ as follows:
  1. Choose $\omega \in_R X$ uniformly at random
  2. Compute $t = \phi(\omega)$
  3. Compute $e_i = H(b_i, t)$ for $i = 1, \ldots, n$
  4. Compute $a = \sum_i e_i a_i$
  5. Compute $c = H(b, t)$ $\Leftarrow$ we don't want P to compute b
  6. Compute $s = \omega + c \cdot a$
  7. Publish $\pi = (t, s)$
- V computes $e_i = H(b_i, t)$, $c = H(b_1, \ldots, b_n, t)$, $b = \prod_i b_i^{e_i}$, and checks $\phi(s) \stackrel{?}{=} t \cdot b^c$

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   2. Compute $t = \phi(\omega)$
   3. Compute $e_i = H(b_i, t)$ for $i = 1, \ldots, n$
   4. Compute $a = \sum_i e_i a_i$
   5. Compute $c = H(b_1, \ldots, b_n, t)$
   6. Compute $s = \omega + c \cdot a$
   7. Publish $\pi = (t, s)$

- V computes $e_i = H(b_i, t)$, $c = H(b_1, \ldots, b_n, t)$, $b = \prod_i b_i^{e_i}$, and checks $\phi(s) \stackrel{?}{=} t \cdot b^c$

Type-1 Batch Proof: Performance

- Proof $\pi = (t, s)$ has size $O(1)$
- Generation: $O(1)$
  - → Let r be the number of modExps in ϕ
  - → Assume that $a = \sum_i e_i a_i$ can be computed efficiently
  - → $modExps(n) = r$
- Verification: $O(n)$
  - → Computing $b = \prod_i b_i^{e_i}$ requires n modExps
  - → Computing $\phi(s)$ requires r modExps
  - → Computing b^c requires 1 modExp
  - → $modExps(n) = n + r + 1$
- Remark: $e_1, \ldots, e_n$ and c are usually small (e.g., 160 bits)

Outline

Type-2 Batch Proof

- Consider n one-way group homomorphisms $\phi_i : X \rightarrow Y$
- Let $b_1, \ldots, b_n$ be publicly known, where $b_i = \phi_i(a)$
- P proves knowledge of a as follows:
  - → V chooses $e_1, \ldots, e_n$ uniformly at random

$$b = \prod_i b_i^{e_i} = \prod_i \phi_i(a)^{e_i} = \prod_i \phi_i(e_i \cdot a)$$

  - → P generates a preimage proof $\pi = (t, s)$ for $b = \phi_e(a)$, where $\phi_e(x) = \prod_i \phi_i(e_i \cdot x)$ is a new one-way group homomorphism
- V computes $b = \prod_i b_i^{e_i}$ and verifies π
- Again, P does not need to compute $b = \prod_i b_i^{e_i}$

Type-2 Batch Proof: Non-Interactive Version

- Consider n one-way group homomorphisms $\phi_i : X \rightarrow Y$
- Let $b_1, \ldots, b_n$ be publicly known, where $b_i = \phi_i(a)$
- P proves knowledge of a as follows:
  1. Choose $\omega \in_R X$ uniformly at random
  2. Compute $t = \phi_e(\omega)$
  3. Compute $e_i = H(b_i, t)$ for $i = 1, \ldots, n$
  4. Compute $c = H(b_1, \ldots, b_n, t)$
  5. Compute $s = \omega + c \cdot a$
  6. Publish $\pi = (t, s)$
- V computes $e_i = H(b_i, t)$, $b = \prod_i b_i^{e_i}$, $c = H(b, t)$, and checks $\phi_e(s) \stackrel{?}{=} t \cdot b^c$

Type-2 Batch Proof: Non-Interactive Version

- Consider n one-way group homomorphisms $\phi_i : X \to Y$
- Let $b_1, \ldots, b_n$ be publicly known, where $b_i = \phi_i(a)$
- P proves knowledge of a as follows:
  1. Choose $\omega \in_R X$ uniformly at random
  2. Compute $t = \phi_e(\omega)$ ⇐ ϕ_e depends on e_i
  3. Compute $e_i = H(b_i, t)$ for $i = 1, \ldots, n$
  4. Compute $c = H(b_1, \ldots, b_n, t)$
  5. Compute $s = \omega + c \cdot a$
  6. Publish $\pi = (t, s)$
- V computes $e_i = H(b_i, t)$, $b = \prod_i b_i^{e_i}$, $c = H(b, t)$, and checks $\phi_e(s) \stackrel{?}{=} t \cdot b^c$

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- P proves knowledge of a as follows:
  1. Choose $\omega \in_R X$ uniformly at random
  2. Compute $e_i = H(b_i, t)$ for $i = 1, \ldots, n$
  3. Compute $t = \phi_e(\omega)$
  4. Compute $c = H(b_1, \ldots, b_n, t)$
  5. Compute $s = \omega + c \cdot a$
  6. Publish $\pi = (t, s)$
- V computes $e_i = H(b_i, t)$, $b = \prod_i b_i^{e_i}$, $c = H(b, t)$, and checks $\phi_e(s) \stackrel{?}{=} t \cdot b^c$

Type-2 Batch Proof: Non-Interactive Version

- Consider n one-way group homomorphisms $\phi_i : X \rightarrow Y$
- Let $b_1, \ldots, b_n$ be publicly known, where $b_i = \phi_i(a)$
- P proves knowledge of a as follows:

  1. Choose $\omega \in_R X$ uniformly at random
  2. Compute $e_i = H(b_i, t)$ for $i = 1, \ldots, n \Leftarrow e_i$ depend on t
  3. Compute $t = \phi_e(\omega)$
  4. Compute $c = H(b_1, \ldots, b_n, t)$
  5. Compute $s = \omega + c \cdot a$
  6. Publish $\pi = (t, s)$

- V computes $e_i = H(b_i, t)$, $b = \prod_i b_i^{e_i}$, $c = H(b, t)$, and checks $\phi_e(s) \stackrel{?}{=} t \cdot b^c$

Type-2 Batch Proof: Non-Interactive Version

- Consider n one-way group homomorphisms $\phi_i : X \rightarrow Y$
- Let $b_1, \ldots, b_n$ be publicly known, where $b_i = \phi_i(a)$
- P proves knowledge of a as follows:
  1. Choose $\omega \in_R X$ uniformly at random
  2. Compute $e_i = H(b_i)$ for $i = 1, \ldots, n$
  3. Compute $t = \phi_e(\omega)$
  4. Compute $c = H(b_1, \ldots, b_n, t)$
  5. Compute $s = \omega + c \cdot a$
  6. Publish $\pi = (t, s)$
- V computes $e_i = H(b_i, t)$, $b = \prod_i b_i^{e_i}$, $c = H(b, t)$, and checks $\phi_e(s) \stackrel{?}{=} t \cdot b^c$

Type-2 Batch Proof: Non-Interactive Version

- Consider n one-way group homomorphisms $\phi_i : X \rightarrow Y$
- Let $b_1, \ldots, b_n$ be publicly known, where $b_i = \phi_i(a)$
- P proves knowledge of a as follows:
  1. Choose $\omega \in_R X$ uniformly at random
  2. Compute $e_i = H(b_i)$ for $i = 1, \ldots, n$ $\Leftarrow$ Is this secure?
  3. Compute $t = \phi_e(\omega)$
  4. Compute $c = H(b_1, \ldots, b_n, t)$
  5. Compute $s = \omega + c \cdot a$
  6. Publish $\pi = (t, s)$
- V computes $e_i = H(b_i, t)$, $b = \prod_i b_i^{e_i}$, $c = H(b, t)$, and checks $\phi_e(s) \stackrel{?}{=} t \cdot b^c$

Type-2 Batch Proof: Performance

- Proof $\pi = (t, s)$ has size $O(1)$
- Generation: $O(n)$
  - → Let $\bar{r}$ be average the number of modExps in ϕ_i
  - → Computing $\phi_e(\omega)$ requires $n \cdot \bar{r}$ modExps
  - → $modExps(n) = n \cdot \bar{r}$
- Verification: $O(n)$
  - → Computing $b = \prod_i b_i^{e_i}$ requires n modExps
  - → Computing $\phi_e(s)$ requires $n \cdot \bar{r}$ modExps
  - → Computing b^c requires 1 modExp
  - → $modExps(n) = n \cdot (\bar{r} + 1) + 1$
- Remark: $e_1, \ldots, e_n$ and c are usually small (e.g., 160 bits)

Recapitulation

		modExps(n)			
		small exponents	regular exponents	total	
Composition	Generation	–	$n \cdot \bar{r}$	$n \cdot \bar{r}$	$O(n)$
	Verification	n	$n \cdot \bar{r}$	$n \cdot (\bar{r} + 1)$	$O(n)$
Type-1	Generation	–	r	r	$O(1)$
	Verification	$n + 1$	r	$n + r + 1$	$O(n)$
Type-2	Generation	–	$n \cdot \bar{r}$	$n \cdot \bar{r}$	$O(n)$
	Verification	$n + 1$	$n \cdot \bar{r}$	$n \cdot (\bar{r} + 1) + 1$	$O(n)$

Outline

Square and Multiply Algorithm (SMA)

- General idea for computing x^e efficiently (in a semigroup)

$$x^e = \begin{cases} 1, & \text{if } e = 0 \\ (x^{e/2})^2, & \text{if } e \text{ is even} \\ x \cdot (x^{(e-1)/2})^2, & \text{if } e \text{ is odd} \end{cases}$$

$$= \begin{cases} 1, & \text{if } e = 0 \\ x^{e \bmod 2} \cdot (x^{\lfloor e/2 \rfloor})^2, & \text{otherwise} \end{cases}$$

- Let $L = \log e$ denote the bit length of e
- SMA uses L multiplications and L squarings (worst case)
- Total multiplications: $2L$

Square and Multiply for Products of Powers

- To compute a product of powers $\prod_{i=1}^{n} x_i^{e_i}$, SMA uses $n{\cdot}L$ multiplications and $n{\cdot}L$ squarings
- $n-1$ multiplications are needed for the final result
- Total multiplications: $2{\cdot}n{\cdot}L + n - 1 = n \cdot (2L+1) - 1$
- SMA for products of power (in a commutative semigroup)

$$\prod_i x_i^{e_i} = \begin{cases} 1, & \text{if } e_1 = \cdots = e_n = 0 \\ \prod_i \left(x_i^{e_i \bmod 2} \cdot (x_i^{\lfloor e_i/2 \rfloor})^2 \right), & \text{otherwise} \end{cases}$$

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- SMA for products of power (in a commutative semigroup)

$$\prod_i x_i^{e_i} = \begin{cases} 1, & \text{if } e_1 = \cdots = e_n = 0 \\ \prod_i x_i^{e_i \bmod 2} \cdot \prod_i (x_i^{\lfloor e_i/2 \rfloor})^2, & \text{otherwise} \end{cases}$$

Square and Multiply for Products of Powers

- To compute a product of powers $\prod_{i=1}^{n} x_i^{e_i}$, SMA uses $n \cdot L$ multiplications and $n \cdot L$ squarings
- $n - 1$ multiplications are needed for the final result
- Total multiplications: $2 \cdot n \cdot L + n - 1 = n \cdot (2L + 1) - 1$
- SMA for products of power (in a commutative semigroup)

$$\prod_i x_i^{e_i} = \begin{cases} 1, & \text{if } e_1 = \cdots = e_n = 0 \\ \prod_i x_i^{e_i \bmod 2} \cdot \left(\prod_i x_i^{\lfloor e_i/2 \rfloor}\right)^2, & \text{otherwise} \end{cases}$$

- Uses $n \cdot L$ multiplications and L squarings
- Total multiplications: $(n + 1) \cdot L$

Outline

Prime Order Co-Domain

- For a batch proof to be sound, ϕ (resp. ϕ_e) must have a prime order co-domain Y
- Otherwise, if $|Y|$ is composite, Y may contain low-order sub-groups
- For example, let $G_2 = \{x_1, x_2\} \subseteq Y$ be a sub-group of Y
- If we pick two integers $e_1, e_2 \in \mathbb{Z}$ at random, then

$$P(x_1^{e_1} = x_2^{e_2}) = \frac{1}{2}$$

- Therefore, the probability that an incorrect input $b_i \in G_2$ can pass the verification is $\frac{1}{2}$

Prime Order Sub-Group of $\mathbb{Z}_p^*$

- To avoid this problem, let the co-domain of ϕ (resp. ϕ_e) be a prime order sub-group $G_q \subset \mathbb{Z}_p^*$, where $p = k \cdot q + 1$
- If we pick two integers $e_1, e_2 \in \mathbb{Z}$ at random, then

$$P(x_1^{e_1} = x_2^{e_2}) = \frac{1}{q}$$

  for any distinct values $x_1, x_2 \in G_q$
- Therefore, batch proofs are sound for high-order sub-groups $G_q \subset \mathbb{Z}_p^*$

Testing for Group Membership

- Great, working with prime order co-domains $G_q \subset \mathbb{Z}_p^*$ seems to work. But what if $b_i \notin G_q$?
- For example, let $p = 2 \cdot q + 1$ be a safe prime. Then $x \in G_q$ implies
  - → $(p - x) \in \mathbb{Z}_n^* \setminus G_q$
  - → $x^2 = (p - x)^2$
  - → $x^e = (p - x)^e$ for $e \geq 2$
- Therefore, V needs to test group membership $b_i \in G_q$ for all public inputs $b_i = \phi(a_i)$ resp. $b_i = \phi_i(a)$
- Testing group membership in $G_q \subset \mathbb{Z}_p^*$ requires one modExp

Recapitulation: Update

		$modExps(n)$			
		small exponents	regular exponents	total	
Composition	Generation	–	$n \cdot \bar{r}$	$n \cdot \bar{r}$	$O(n)$
	Verification	n	$n \cdot \bar{r}$	$n \cdot (\bar{r} + 1)$	$O(n)$
Type-1	Generation	–	r	r	$O(1)$
	Verification	$n + 1$	$n + r$	$2n + r + 1$	$O(n)$
Type-2	Generation	–	$n \cdot \bar{r}$	$n \cdot \bar{r}$	$O(n)$
	Verification	$n + 1$	$n \cdot (\bar{r} + 1)$	$n \cdot (\bar{r} + 2) + 1$	$O(n)$

Outline

Conclusion

- There are two types of generic batch proofs
- Type-1 Proof: Common Function
  - → Proof size: $O(1)$
  - → Proof generation runs in $O(1)$ time
  - → Verification not significantly improved
  - → Examples: multi-commitment, multi-encryption, etc.
- Type-2 Proof: Common Preimage
  - → Proof size: $O(1)$
  - → Proof generation and verification not significantly improved
  - → Examples: multi-decryption, multi-blinding, etc.
- Caution: proofs work only for prime order co-domains

Open Quesions

- Missing security proof for the generic proof construction
- Dependency problem in the non-interactive Type-2 Proof
- Missing security proof for the non-interactive version (Fiat-Shamir, random oracle model)
- Implementation in UniCrypt
- Useful for UniVote?